Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

A Note on the General Instability of Eccentrically Stiffened Cylinders

GEORGE J. SIMITSES*
Georgia Institute of Technology, Atlanta, Ga.

Nomenclature

A_x, A_y	= stringer and ring cross-sectional area, in.^{2}
D_{xx}, D_{yy}, D_{xy}	= orthotropic flexural and twisting stiffnesses, in.-lb
E, E_x, E_y	= Young's modulus of elasticity, psi
e	= stringer and ring eccentricities, in.
$\bar{e}$	= nondimensionalized eccentricities [$= \pi^2 Re/L^2$]
E_{xx}, E_{yy}	= orthotropic extensional stiffnesses, lb/in.
$(GJ)_{x \text{ or } y}$	= stiffener contributions to the torsional stiffness, in.^{2}-lb
G_{xy}	= in-plane skin shear stiffness, lb/in.
h	= skin thickness, in.
I_{xc}, I_{yc}	= stiffener moment of inertia about centroidal axes, in.^{4}
k_x, k_y, k_s	= applied load coefficients
L	= cylinder length, in.
l_x, l_y	= stiffener spacings, in.
m	= number of half-sine waves in x direction
M_x, M_y, M_{xy}	= moment resultants, lb
n	= number of full-sine waves in y direction
N_x^0, N_{xy}^0, N_y^0	= membrane stress resultants, lb/in.
q	= applied pressure (positive outward), psi
R	= cylinder radius (to skin midsurface), in.
T	= applied torque, in.-lb
u, v, w	= displacement components of reference surface points, in.
Z_t	= curvature parameter [$= \{(1 - \nu^2)E_{xx}L^4/12R^2D_{yy}\}^{1/2}$]
λ_{ij}, ρ_{ij}	= extensional and flexural stiffness parameters, Eq. (6c)
ν	= Poisson's ratio

INVESTIGATIONS of general instability of eccentrically stiffened cylinders under the action of single load application (uniform axial compression, uniform lateral or hydrostatic pressure, and torsion) have been reported by a number of authors.[1-4] Most of these authors have used orthotropic thin shell theory and have reduced the problem to an eigenvalue boundary problem, with three differential equations. Using the geometry and sign convention shown in Figs. 1 and 2, these three governing equations are

$$\left[E_{xx}\frac{\partial^2}{\partial x^2} + G_{xy}\frac{\partial^2}{\partial y^2}\right]u^1 + \left[(G_{xy} + \nu E_{yyp})\frac{\partial^2}{\partial x \partial y}\right]v^1 = \left[\left(q - \frac{\nu}{R}E_{yyp}\right)\frac{\partial}{\partial x} + e_x E_{xxst}\frac{\partial^3}{\partial x^3}\right]w^1 \quad \text{(1a)}$$

$$\left[(G_{xy} + \nu E_{xxp})\frac{\partial^2}{\partial x \partial y}\right]u^1 + \left[E_{yy}\frac{\partial^2}{\partial y^2} + G_{xy}\frac{\partial^2}{\partial x^2}\right]v^1 = \left[\left(q - \frac{1}{R}E_{yy}\right)\frac{\partial}{\partial y} + e_y E_{yyr}\frac{\partial^3}{\partial y^3}\right]w^1 \quad \text{(1b)}$$

$$\left[(D_{xx} + e_x^2 E_{xxst})\frac{\partial^4}{\partial x^4} + 2\left(D_{xy} + \frac{\nu}{2}D_{xxp} + \frac{\nu}{2}D_{yyp}\right) \times \frac{\partial^4}{\partial x^2 \partial y^2} + (D_{yy} + e_y^2 E_{yyr})\frac{\partial^4}{\partial y^4} + \frac{E_{yy}}{R^2} - 2\frac{e_y}{R}E_{yyr}\frac{\partial^2}{\partial y^2}\right]w^1 + \left[\frac{\nu}{R}E_{xxp}\frac{\partial}{\partial x} - e_x E_{xxst}\frac{\partial^3}{\partial x^3}\right]u^1 + \left[\frac{E_{yy}}{R}\frac{\partial}{\partial y} - e_y E_{yyr}\frac{\partial^3}{\partial y^3}\right]v^1 = N_x^0 w_{,xx}^1 + N_y^0 w_{,yy}^1 + 2N_{xy}^0 w_{,xy}^1 \quad \text{(1c)}$$

where

$$E_{xxp}\dagger = E_{yyp}\dagger = Eh/(1 - \nu^2)$$

$$D_{xxp}\dagger = D_{yyp}\dagger = D = Eh^3/12(1 - \nu^2)$$

$$E_{xxst} = E_x A_x/l_x \qquad E_{yyr} = E_y A_y/l_y$$

$$G_{xy} = Eh/2(1 + \nu)$$

$$D_{xxst} = E_x I_{xc}/l_x \qquad D_{yyr} = E_y I_{yc}/l_y \quad \text{(2)}$$

$$E_{xx} = E_{xxp} + E_{xxst}$$

$$E_{yy} = E_{yyp} + E_{yyr} \qquad D_{xx} = D_{xxp} + D_{xxst}$$

$$D_{yy} = D_{yyp} + D_{yyr}$$

$$D_{xy} = D(1 - \nu) + (GJ)_x/2l_x + (GJ)_y/2l_y$$

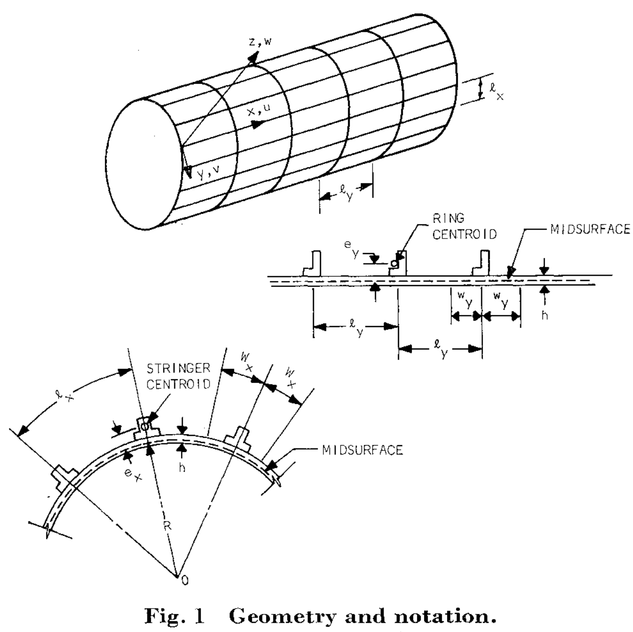

Fig. 1 Geometry and notation.

† These two need not be equal if skin wrinkling is present. They may be reduced by the effective width (see Fig. 1) or some other scheme.

Received November 16, 1966; revision received May 5, 1967. The work was sponsored by the Lockheed-Georgia Company, and it was performed at its Marietta, Ga. plant. [6.02, 6.04]

* Associate Professor of Aerospace Engineering; also Consultant, Lockheed-Georgia Company, Department 72-26, Marietta, Ga. Member AIAA.

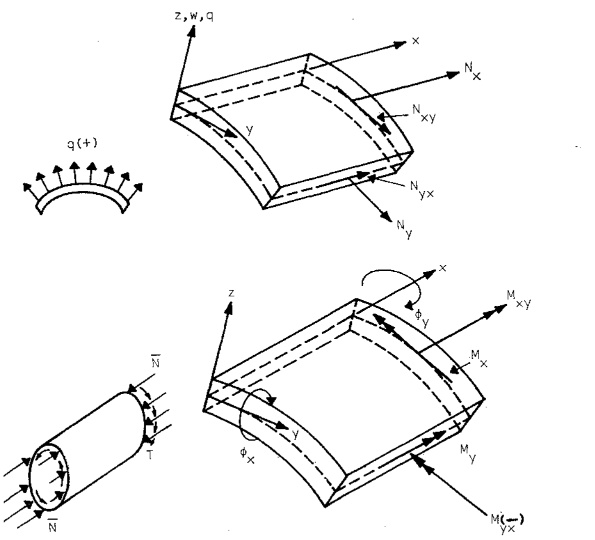

Fig. 2 Sign convention.

Note that E_x and E_y are the stiffener Young's moduli, A_i and I_{ic} the stiffener areas and moments of inertia about centroidal axes. Furthermore, it should be noted that Eqs. (1) were derived under the assumption that the pressure load q remains normal to the deflected midsurface during the buckling process.

The eigenvalues for the problem are

$$N_y{}^0 = qR \qquad N_x{}^0 = \tfrac{1}{2}qR\ddagger - \bar{N} \qquad N_{xy}{}^0 = T/2\pi R^2 \tag{3}$$

Since the operators in Eqs. (1) are commutative, it is possible to derive a single higher order Donnell-Batdorf type of equation by eliminating u^1 and v^1. This higher order equation is

$$\nabla_D w^1 + \nabla_E{}^{-1}\,[12Z_t{}^2/\pi^4(1-\nu^2)\nabla_C w^1 - k_y(L/\pi R)^2\nabla_P w^1] = (L/\pi)^2[(\tfrac{1}{2}k_y - k_x\rho_{xx})w_{,xx}{}^1 + k_y w_{,yy}{}^1 + 2k_s w_{,xy}{}^1] \tag{4}$$

where

$$\nabla_E = \left(\frac{L}{\pi}\right)^4\left[\frac{\partial^4}{\partial x^4} + \left(\frac{\lambda_{yy}}{\lambda_{xy}} - \nu\lambda_{xxp} - \nu\lambda_{yyp} - \nu^2\,\frac{\lambda_{xxp}\lambda_{yyp}}{\lambda_{xy}}\right)\frac{\partial^4}{\partial x^2\partial y^2} + \lambda_{yy}\frac{\partial^4}{\partial y^4}\right] \tag{5a}$$

$$\nabla_D = \left(\frac{L}{\pi}\right)^4\left[\rho_{xx}\frac{\partial^4}{\partial x^4} + 2\left(\rho_{xy} + \frac{\nu}{2}\,\rho_{xxp} + \frac{\nu}{2}\,\rho_{yyp}\right)\times\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}\right] \tag{5b}$$

$$\begin{aligned}\nabla_C = \left(\frac{L}{\pi}\right)^8\Bigg[&\bar{e}_x{}^2\lambda_{xxst}\lambda_{xxp}\frac{\partial^8}{\partial x^8} + \bar{e}_x{}^2\lambda_{xxst}\left(\lambda_{xxp}\frac{\lambda_{yy}}{\lambda_{xy}} - \nu\lambda_{xxp} - \nu\lambda_{yyp} - \nu^2\,\frac{\lambda_{xxp}\lambda_{yyp}}{\lambda_{xy}}\right)\frac{\partial^8}{\partial x^6\partial y^2} + \Bigg\{\bar{e}_x{}^2\lambda_{xxst}\lambda_{yy} + \\ &\bar{e}_x\bar{e}_y\lambda_{xxst}\lambda_{yyr}\left(2 + \nu\,\frac{\lambda_{xxp}+\lambda_{yyp}}{\lambda_{xy}}\right) + \bar{e}_y{}^2\lambda_{yyr}\Bigg\}\times\frac{\partial^8}{\partial x^4\partial y^4} + \bar{e}_y{}^2\lambda_{yyr}\left(\frac{\lambda_{yyp}}{\lambda_{xy}} - \nu\lambda_{xxp} - \nu\lambda_{yyp} - \nu^2\,\frac{\lambda_{xxp}\lambda_{yyp}}{\lambda_{xy}}\right)\times \\ &\frac{\partial^8}{\partial x^2\partial y^6} + \bar{e}_y{}^2\lambda_{yyr}\lambda_{yyp}\frac{\partial^8}{\partial y^8} + \nu\bar{e}_x\lambda_{xxst}(\lambda_{xxp}+\lambda_{yyp})\frac{\partial^6}{\partial x^6} - 2(\bar{e}_x\lambda_{xxst} + \bar{e}_y\lambda_{yyr})\frac{\partial^6}{\partial x^4\partial y^2} + \nu\bar{e}_y\lambda_{yyr}(\lambda_{xxp}+\lambda_{yyp})\times \\ &\frac{\partial^6}{\partial x^2\partial y^4} + (\lambda_{yy} - \nu^2\lambda_{xxp}\lambda_{yyp})\frac{\partial^4}{\partial x^4}\end{aligned} \tag{5c}$$

$$\begin{aligned}\nabla_P = \left(\frac{L}{\pi}\right)^6\Bigg[&\bar{e}_x\lambda_{xxst}\left\{\frac{\partial^6}{\partial x^6} + \left(\frac{\lambda_{yy}}{\lambda_{xy}} - 1 - \nu\,\frac{\lambda_{yyp}}{\lambda_{xy}}\right)\times\frac{\partial^6}{\partial x^4\partial y^2}\right\} + \bar{e}_y\lambda_{yyr}\left(\frac{1}{\lambda_{xy}} - 1 - \nu\,\frac{\lambda_{xxp}}{\lambda_{xy}}\right)\frac{\partial^6}{\partial x^2\partial y^4} + \\ &\bar{e}_y\lambda_{yyr}\frac{\partial^6}{\partial y^6} - \nu\left(\frac{\pi}{L}\right)^2\lambda_{xxp}\frac{\partial^4}{\partial x^4} + \left\{\lambda_{yy}\left(1 - \frac{1}{\lambda_{xy}}\right) + \nu\lambda_{xxp}\left(1 + \nu\,\frac{\lambda_{yyp}}{\lambda_{xy}}\right)\right\}\left(\frac{\pi}{L}\right)^2\frac{\partial^4}{\partial x^2\partial y^2} - \lambda_{yy}\left(\frac{\pi}{L}\right)^2\frac{\partial^4}{\partial y^4}\Bigg]\end{aligned} \tag{5d}$$

$$k_x = NL^2/\pi^2 D_{xx} \qquad k_y = qRL^2/\pi^2 D_{yy} \qquad k_s = N_{xy}{}^0 L^2/\pi^2 D_{yy} \tag{6a}$$

$$Z_t = [(1-\nu^2)L^4E_{xx}/12R^2D_{yy}]^{1/2} \qquad \bar{e} = (\pi R/L)^2(e/R) \tag{6b}$$

$$\lambda_{ij} = E_{ij}\ \text{or}\ G_{ij}/E_{xx} \qquad \rho_{ij} = D_{ij}/D_{yy} \tag{6c}$$

Note that 1) $\lambda_{xxp} + \lambda_{xxst} = 1$, $\lambda_{yyp} + \lambda_{yyr} = \lambda_{yy}$; 2) for isotropic cylinders $Z_t = (L^2/Rh)(1-\nu^2)^{1/2}$, $\lambda_{xxp} = \lambda_{yyp} = \lambda_{yy} = 1$, $\lambda_{xxst} = \lambda_{yyr} = 0$, $\lambda_{xy} = (1-\nu)/2$, $\rho_{xx} = \rho_{xxp} = \rho_{yyp} = 1$, and $\rho_{xy} = (1-\nu)$. For any combination of loads, and either simply supported or clamped boundary conditions, it is possible to arrive at a critical condition by employing a Galerkin procedure (for details see Ref. 5). For some special cases though, a closed-form solution may be obtained. Two such cases are discussed below.

1. Uniform Axial Compression

Under the assumption of axisymmetric buckling ($\beta = nL/\pi R = 0$) and under simple supported boundary conditions the characteristic equation is

$$k_x = m^2 + \frac{12Z_t{}^2}{(1-\nu^2)\pi^4\rho_{xx}}\left[\bar{e}_x{}^2\lambda_{xxst}\lambda_{xxp}m^2 - \nu\bar{e}_x\lambda_{xxst}(\lambda_{xxp}+\lambda_{yyp}) + \frac{1}{m^2}(\lambda_{yy} - \nu^2\lambda_{xxp}\lambda_{yyp})\right] \tag{7}$$

Minimization with respect to m yields

$$k_{x_{cr}} = \left(\frac{48}{1-\nu^2}\right)^{1/2}\frac{Z_t}{\pi^2\rho_{xx}{}^{1/2}}(\lambda_{yy} - \nu^2\lambda_{xxp}\lambda_{yyp})^{1/2}\times\left[1 + \frac{12Z_t{}^2}{(1-\nu^2)\pi^4}\,\frac{\bar{e}_x{}^2\lambda_{xxst}\lambda_{xxp}}{\rho_{xx}}\right]^{1/2} - \frac{12Z_t{}^2}{(1-\nu^2)\pi^4}\,\frac{\nu\bar{e}_x\lambda_{xxst}(\lambda_{xxp}+\lambda_{yyp})}{\rho_{xx}} \tag{8}$$

Note that Eq. (8), when applied to isotropic unstiffened cylinders, reduces to the well-known equation

$$k_{x_{cr}} = 0.702Z \qquad \text{where} \qquad Z = L^2/Rh(1-\nu^2) \tag{9}$$

Finally, it is seen through Eq. (8) that for axisymmetric buckling, the strongest configuration corresponds to placing the stringers on the inside.

2. Uniform Pressure (Lateral Hydrostatic)

If one assumes that the geometry is such that when buckling occurs $\beta^2 \gg 1$ (moderate length cylinders; see Becker[6]), then the characteristic equation becomes

$$-k_y = \beta^2 + \frac{12Z_t{}^2}{(1-\nu^2)\pi^4\lambda_{yy}}\left[\bar{e}_y{}^2\lambda_{yyr}\lambda_{yyr}\beta^2 - \frac{\nu}{\beta^2}\,\bar{e}_y\lambda_{yyr}\times(\lambda_{xxp}+\lambda_{yyp}) + \frac{1}{\beta^6}(\lambda_{yy} - \nu^2\lambda_{xxp}\lambda_{yyp})\right] \tag{10}$$

Equation (10) is applicable to both lateral and hydrostatic pressure, and it is written for $m = 1$. Furthermore, an order analysis yields that the effect of load behavior during the buckling process is negligible for these geometries ($\beta^2 \gg$

‡ In the case of lateral pressure, this term is set to zero.

1). Minimization with respect to β yields

$$k_{y_{cr}} = \frac{4}{3}\frac{C(2A)^{3/2}}{[(B^2 + 4AC) - B]^{3/2}} - \frac{B(2A)^{1/2}}{[(B + 4AC) - B]^{1/2}} \tag{11}$$

where

$$A = [1 + 12Z_t^2\acute{e}_y^2\lambda_{yy_r}\lambda_{yy_p}/(1 - \nu^2)\pi^4\lambda_{yy}]$$

$$B = 12Z_t^2\nu\acute{e}_y\lambda_{yy_r}(\lambda_{xx_p} + \lambda_{yy_p})/(1 - \nu^2)\pi^4\lambda_{yy}$$

$$C = 36Z_t^2(1 - \nu^2\lambda_{xx_p}\lambda_{xx_p}/\lambda_{yy})/(1 - \nu^2)\pi^4$$

Note that B could be either positive or negative depending on the outside or inside positioning of the rings. Furthermore, if $\bar{e}_y = 0$ then $B = 0$, $A = 1$, and Eq. (11) reduces to

$$k_{y_{cr}} = 1.04Z_t^{1/2}[(1 - \nu^2\lambda_{xx_p}\lambda_{yy_p}/\lambda_{yy})/(1 - \nu^2)]^{1/4} \tag{12}$$

Finally, when the assumption of isotropy is made, Eq. (10) reduces to the well-known expression

$$-k_{y_{cr}} = 1.04Z \tag{13}$$

References

[1] Block, D. L., Card, M. F., and Mikulas, M. M., Jr., "Buckling of eccentrically stiffened orthotropic cylinders," NASA TN D-2960 (1965).

[2] Baruch, M. and Singer, J., "Effect of eccentricity of stiffeners on the general instability of stiffened cylindrical shells under hydrostatic pressure," J. Mech. Eng. Sci. **5**, 23–27 (1963).

[3] Hedgepeth, I. M. and Hall, D. B., "Stability of stiffened cylinders," AIAA Paper 65-79 (1965); also AIAA J. **3**, 2275–2286 (1965).

[4] Deluzio, A. and Stuhlman, C., "Influence of stiffener eccentricity and end moments on cylinder compression stability," Lockheed Missiles & Space Co., LMSC A-804608 (1964).

[5] Simitses, G. J., "General instability of eccentrically stiffened cylinders under combined loads," Lockheed-Georgia Co., Marietta, Ga., ER 8562 (1966).

[6] Becker, H., "General instability of stiffened cylinders," NACA TN 4237 (1958).

Aircraft Systems Studies under Long-Term World Environmental Storage

A. B. Billet*
Vickers Inc., Troy, Mich.

Introduction

WITH the introduction of intercontinental ballistic missiles for field use, the importance of long-term storage effects on these missiles and their operating systems has become quite significant. It is necessary, of course, that these weapons be stored in their concrete operational silos on a ready-to-go basis for periods of five or more years.

The military personnel and technical staffs using these vehicles have raised the question of what effects this long-term storage has on missile flight control systems, their components and parts. Of particular concern was the effect on hydraulic fluids, "O" rings, and servovalve subassemblies. To help answer some of these questions and aid the preparation of military storage specifications, the Society of Automotive Engineers Long-Term Storage Panel was organized in 1959 with the author as chairman.

Presented as Paper 67-185 at the AIAA 5th Aerospace Sciences Meeting, New York, January 23–26, 1967; submitted January 16, 1967; revision received May 24, 1967. [10.01]

* Executive Engineer. Member AIAA.

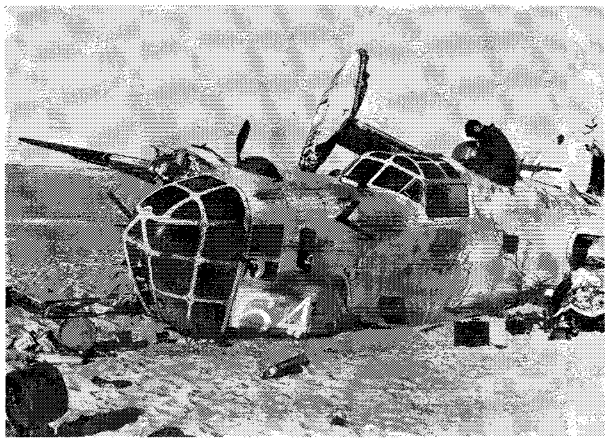

Fig. 1 "Lady Be Good" aircraft—African desert.

The problem at the outset of any long-term storage study, obviously, is acquisition of study material. Controlled experiments can be set up, but even under accelerated conditions, these take time.

As part of the investigations of this panel, hydraulic systems and hydraulic components from certain World War II types of aircraft were evaluated, with the data then being extrapolated for present-day systems. Many components were removed from aircraft at Davis-Monthan Air Base in Arizona for study, where they had been stored in the open desert, by the Air Force, for 5-to-10 yr periods of time. It was no surprise that the hydraulic equipment, instrumentation, and various other items of equipment proved to be in excellent condition. At this point, luck entered the picture.

Desert Environmental Investigations

In 1960, a study was made of 16 pieces of equipment removed from the B-24 aircraft "Lady Be Good" (Fig. 1) that had lain in the North African desert for 17 years. The environment was generally warm and dry—excellent for the equipment, as our investigations subsequently proved. The Vickers engine-driven pump and turret retraction motor met the requirements of new units, following this 17-yr storage period.

"Lady Be Good" had spent her last 17 years in a solar oven, baking at air temperatures of 120°F in the summer. Equipment inside the plane probably had been heated to more than 200°F. As far as her hydraulic system was concerned, this intense heat had seemingly provided a near-perfect environment, with one exception; the Buna-N diaphragm of the accumulator had slightly stiffened and had caused the unit to lose its gaseous charge.

Arctic Environmental Investigations

When the Air Force rediscovered the B-17 aircraft "My Gal Sal" in the middle of the Greenland icecap (Fig. 2) in October 1964, it was believed that this would be an excellent situation

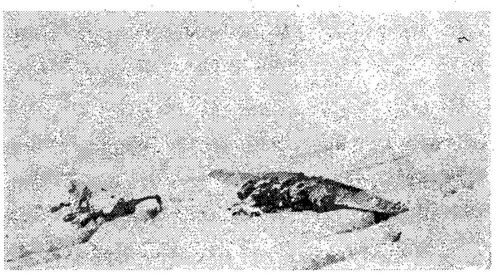

Fig. 2 "My Gal Sal" on Greenland icecap.